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DUBINS VEHICLE WITH MOTION AND COMMUNICATION CONSTRAINTS

Rishi Bajpai
JEC Jabalpur, India

ABSTRACT

In this paper an algorithm has been presented based on minimum-time motion planning and routing problems for the Dubins vehicle that is constrained to move along planar paths of bounded curvature, without reversing direction. The problem is motivated by autonomous aerial vehicle applications. The worst-case length that grows linearly with n and proposed a novel algorithm with performance within a constant factor of the optimum for the worst-case point sets. In doing this, obtain an upper bound on the optimal length in the classical point-to-point problem. Second, a stochastic version of the DTSP where the n targets are randomly and independently sampled from a uniform distribution. The expected length of such a tour is of order at least n 2/3 and proposes a novel algorithm yielding a solution with length of order n 2/3 with probability one. A dynamic version of the DTSP: given a stochastic process that generates target points, is there a policy that guarantees that the number of unvisited points does not diverge over time. The expected wait time of the proposed stabilizing algorithms is probably within a constant factor from the optimum.

Keywords- *Holonomic vehicle, Dubins vehicle, Uninhabited aerial vehicles, robotics.*

I. INTRODUCTION

In this paper a novel class of optimal motion planning problems for a nonholonomic vehicle required to visit collections of points in the plane is simulated, where the vehicle is said to *visit* a region in the plane if the vehicle goes to that region and passes through it [1, 2]. This class of problem has two main ingredients. First, the robot model is the so-called Dubins vehicle, namely, a nonholonomic vehicle that is constrained to move along paths of bounded curvature without reversing direction. Second, the objective is to find the shortest path for such vehicle through a given set of target points. Except for the nonholonomic constraint, this task is akin to the classic Traveling Salesperson Problem (TSP) and in particular to the Euclidean TSP (ETSP), in which the shortest path between any two target locations is a straight segment. In summary, the focus of this paper is the analysis and the algorithmic A practical motivation to study the DTSP arises naturally in robotics and Uninhabited Aerial Vehicles (UAVs) applications [3]. DTSP algorithms to the setting of a UAV monitoring a collection of spatially distributed points of interest were applied. In one scenario, the location of the points of interest might be known and static. Additionally, UAV applications motivate the study of the Dynamic Traveling Repairperson Problem (DTRP), in which the UAV is required to visit a dynamically changing set of targets. Such problems are examples of distributed task allocation problems and are currently generating much interest; e.g., [4] discusses complexity issues related to UAVs assignments problems, [5] considers Dubins vehicles keeping under surveillance multiple mobile targets, [6] considers missions with dynamic threats; other relevant works include [7],. The literature on the Dubins vehicle is very rich and includes contributions from researchers in multiple disciplines. The minimum-time point-to-point path-planning problem with bounded curvature was originally introduced by Markov [8] and a first solution was given by Dubins. Modern treatments on point-to-point planning exploit the minimum principle [9], carefully account for symmetries in the problem, and consider environments with obstacles [10]. The Dubins vehicle is commonly accepted as a reasonably accurate kinematic model for aircraft motion planning problems and its study is included in recent texts [11-12]. The TSP and its variations continue to attract great interest from a wide range of fields, including operations research, mathematics and computer science. Tight bounds on the asymptotic dependence of the ETSP on the number of targets are given in the early work [13] and in the survey [14]. Exact algorithms, heuristics as well as polynomial-time constant factor approximation algorithms are available for the Euclidean TSP, Based on our earlier works, this paper presents a comprehensive treatment of the worst-case DTSP, the DTSP in the stochastic setting, and the DTRP. The paper is organized as follows. Section II contains the problem formulation. The worst-case DTSP algorithm is treated in Section III. Section IV contains the treatment of the DTSP for the stochastic setting. Section V contains the treatment of the dynamic version of the DTSP. conclusion with a few remarks about future work presented in Section VI.

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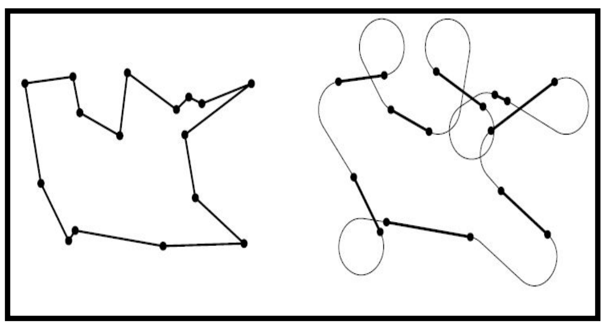
II. PROBLEM SETUP: FROM THE EUCLIDEAN TO THE DUBINS TRAVELING SALESPERSON PROBLEM

In this section we setup the main problem and basic notations. A Dubins vehicle is a planar vehicle that is constrained to move along paths of bounded curvature, without reversing direction and maintaining a constant speed. Accordingly, we define a feasible curve for the Dubins vehicle or a Dubins path, as a curve $\gamma: [0, T] \to R^2$ that is twice differentiable almost everywhere, and such that the magnitude of its curvature is bounded above by $1/\rho$, where $\rho > 0$ is the minimum turning radius. We also let Length(γ) = $\int_0^T \|\gamma'(t)\| dt$ be the length of a differentiable curve $\gamma : [0, T] \to R^2$. We represent the vehicle configuration by the triplet $(x, y, \psi) \in SE(2)$, where (x, y) are the Cartesian coordinates of the vehicle and ψ is its heading. Let $P = \{p_1, \ldots, p_n\}$ be a set of n points in a compact region $Q \subset R^2$ and P_n be the collection of all point sets $P \subset Q$ with cardinality n. Let ETSP(P) denote the cost of the Euclidean TSP over P, i.e., the length of the shortest closed path through all points in P. Correspondingly, let $DTSP_\rho(P)$ denote the cost of the Dubins TSP over P, i.e., the length of the shortest closed Dubins path through all points in P with minimum turning radius ρ.

The algorithm consists of the following steps:

(i) set $(a_1, \ldots, a_n) :=$ optimal ETSP ordering of P
(ii) set $\psi_1 :=$ orientation of segment from a_1 to a_2
(iii) for $i \in \{2, \ldots, n-1\}$, do
if i is even, then set $\psi_i := \psi_{i-1}$, else set $\psi_i :=$ orientation of segment from a_i to a_{i+1}
(iv) if n is even, then set $\psi_n := \psi_{n-1}$, else set $\psi_n :=$ orientation of segment from a_n to a_1
(v) return the sequence of configurations $\{(a_i, \psi_i)\}_{i \in \{1,\ldots,n\}}$.
The output of the Algorithm in Figure 1.



Left figure: a graph representing the solution of ETSP over a given P. Right figure: a graph representing the solution given by the Alternating Algorithm on P where the alternate segments of ETSP are retained.

The performance of the Algorithm to obtain an upper bound on DTSPρ(P) and then show that the algorithm performs within a constant factor of the optimal in the worst case. To obtain an upper bound on the length of the Dubins vehicle while executing the Alternating Algorithm, we first obtain an upper bound on the optimal point-to-point problem for the Dubins vehicle. Given an initial configuration $(x_{initial}, y_{initial}, \psi_{initial})$ and a final configuration $(x_{final}, y_{final}, \psi_{final})$, find an upper bound on the length of the shortest Dubins path going from initial to final configuration.

III. THE RECURSIVE BEAD-TILING ALGORITHM

In this section, we design a novel algorithm that computes a Dubins path through a point set in Q. The proposed algorithm consists of a sequence of phases; during each phase, a Dubins tour (i.e., a closed path with bounded curvature) is constructed that "sweeps" the set Q. We begin by considering a tiling of the plane such that $Area(B_\rho(\ell)) = WH/(2n)$; in such a case, $\mu(\ell(n)) = 1/(2n)$,

$$\ell(n) = 2\left(\frac{\rho W H}{n}\right)^{\frac{1}{3}} + o\left(n^{-\frac{1}{3}}\right), \qquad (n \to +\infty)$$

In the first phase of the algorithm, a Dubins tour is constructed with the following properties:

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(i) it visits all non-empty beads once,
(ii) (ii) it visits all rows2 in sequence top-to-down, alternating between left-to-right and right-to-left passes, and visiting all non-empty beads in a row,
(iii) (iii) when visiting a non-empty bead, it services at least one target in it.

In order to visit the targets outstanding after the first phase, a second phase is initiated. Instead of considering single beads, we now consider "meta-beads" composed of two beads each, as shown in Figure 2, and proceed in a way similar to the first phase, i.e., a Dubins tour is constructed with the following properties:

(i) the tour visits all non-empty meta-beads once,
(ii) it visits all (meta-bead) rows in sequence top-to-down, alternating between left- to-right and right-to-left passes, and visiting all non-empty meta-beads in a row,
(iii) when visiting a non-empty meta-bead, it services at least one target in it.

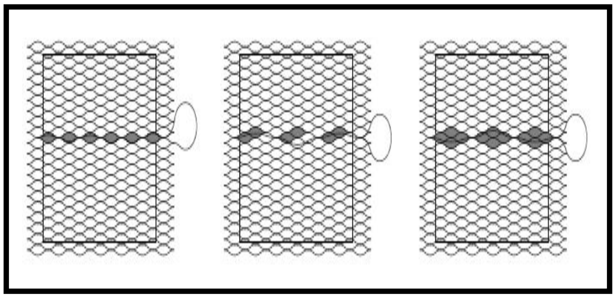

Figure 2: Sketch of "meta-beads" at successive phases in the recursive bead tiling algorithm.

From left to right: phase 1, phase 2 and phase 3. This process is iterated $\log_2 n$ times, and at each phase, meta-beads composed of two neighboring meta-beads from the previous phase are considered; in other words, the meta-beads at the i^{th} phase are composed of 2^{i-1} neighboring beads. After the last recursive phase, the leftover targets are visited using the Alternating Algorithm.

an upper bound on the length of Dubins path as given by the Recursive Bead-Tiling Algorithm is calculated. By comparing this upper bound with the lower bound established earlier, we will conclude that the algorithm provides a constant factor approximation to the optimal stochastic DTSP with high probability.

IV. THE DTRP FOR A SINGLE VEHICLE

The vehicle and sensing model and the DTRP definition is described. The key aspect of the DTRP is that the Dubins vehicle is required to visit a dynamically growing set of targets, generated by some stochastic process. We assume that the Dubins vehicle has unlimited range and target-servicing capacity and that it moves at a unit speed with minimum turning radius $\rho > 0$. Information about the outstanding targets representing the demand at time t is described by a finite set of positions $D(t) \subset Q$, with $n(t) := \text{card}(D(t))$. Targets are generated, and inserted into D, according to a homogeneous (i.e., time-invariant) spatio-temporal Poisson process, with time intensity $\lambda > 0$, and uniform spatial density inside the rectangle Q of width W and height H. In other words, given a set $S \subseteq Q$, the expected number of targets generated in S within the time interval $[t, t']$ is

$$\mathrm{E}\left[\text{card}(D(t') \cap \mathcal{S}) - \text{card}(D(t) \cap \mathcal{S})\right] = \lambda(t' - t)\,\text{Area}(\mathcal{S})$$

(Strictly speaking, the above equation holds when targets are not being removed from the queue D.) Servicing of a target and its removal from the set D, is achieved when the Dubins vehicle moves to the target position. A feedback control policy for the Dubins vehicle is a map assigning a control input to the vehicle as a function of its configuration and of the current outstanding targets. We also consider policies that compute a control input based on a snapshot of the outstanding target configurations at certain time sequences. Let $T = \{t_k\}_{k \in N}$ be a strictly increasing sequence of times at which such computations are started: with some abuse of terminology, we will say that is a receding horizon strategy if it is based on the most recent target data $D_{rh}(t)$, where

$$D_{\text{rh}}(t) = D(\max\{t_{\text{rh}} \in \mathcal{T}_\Phi \mid t_{\text{rh}} \le t\})$$

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The (receding horizon) policy is a stable policy for the DTRP if, under its action

$$n_\Phi = \lim_{t\to+\infty} \mathrm{E}[n(t)|\, \dot{p} = \Phi(p, D_{\mathrm{rh}})] < +\infty$$
,

that is, if the Dubins vehicle is able to service targets at a rate that is, on average, at least as fast as the rate at which new targets are generated. Let T_j be the time that the j^{th} target spends within the set D, i.e., the time elapsed from the time the j^{th} target is generated to the time it is serviced. If the system is stable, then we can write the balance equation (known as Little's formula [15]:

$$n_\Phi = \lambda T_\Phi$$
,

where T := $\lim_{j\to+\infty}$ E[T_j] is the steady-state system time for the DTRP under the policy . Our objective is to minimize the steady-state system time, over all possible feedback control policies, i.e.,

$$T_{\mathrm{DTRP}} = \inf\{T_\Phi \mid \Phi \text{ is a stable control policy}\}$$

Lower and constructive upper bounds

In what follows, we design a control policy that provides a constant-factor approximation of the optimal achievable performance. Consistently with the theme of the chapter, we consider the case of heavy load, i.e., the problem as the time intensity $\lambda \to +\infty$. We first review from [16] a lower bound for the system time, and then present a novel approximation algorithm providing an
upper bound on the performance.

V. THE DTRP FOR MULTIPLE VEHICLES

The DTRP problem that was introduced in the earlier section for a single vehicle can be easily extended to the multiple vehicle case. In this section, We first obtain a lower bound for the system time for m homogeneous Dubins vehicles, and then present a novel strategy providing an upper bound on the performance.

Theorem 1 (Lower bound on the system time for single-vehicle DTRP) For any $\rho > 0$, the system time TDTRP,m for the DTRP for m vehicles in a rectangle of width W and height H satisfies

$$\lim_{\lambda\to+\infty} \frac{T_{\mathrm{DTRP,m}}}{\lambda^2} \geq \frac{81}{64}\frac{\rho W H}{m^3}.$$

Proof. Let us assume that a stabilizing policy is available. In such a case, the number of outstanding targets approaches a finite steady-state value, n^* , related to the system time by Little's formula, i.e., $n^* = \lambda T_{DTRP,m}$. In order for the policy to be stabilizing, the time needed, on average, to service m targets must be no greater than the average time interval in which m new targets are generated. Since there are m vehicles, the average time needed for them to service one target each, in parallel, is no greater than the expected minimum distance (in the Dubins' sense) from an arbitrarily placed vehicle to the closest target; in other words, we can write the stability condition $E[\delta^* (n^*)] \leq m/\lambda$. A bound on the expected value of δ^* has been computed in [16], yielding

$$\frac{3}{4}\left(\frac{3\rho W H}{n^*}\right)^{1/3} \leq \mathrm{E}[\delta^*(n^*)] \leq m\lambda$$

Using Little's formula $n* = \lambda T_{DTRP,m,}$ and rearranging, we get the desired result.

From Theorem 1, one can infer that the system time depends on (i) the area of the region assigned to a vehicle, and (ii) on the shape of the region. In particular, the system time is minimized, for a given area, when one of its dimensions is maximized. This suggests the following strategy for partitioning the environment Q, that we call the Strip-Tiling Algorithm. Partition Q into m strips of width W and height H/m and assign each strip to a vehicle. Tile each strip with beads of length $\ell := \min\{mC_{BTA}/\lambda, 4\rho\}$. Let each vehicle execute the Bead-Tiling Algorithm inside the assigned strip. Then, the following holds:

Theorem 2 (System time for the Strip-Tiling Algorithm) For any $\rho > 0$, $\lambda > 0$ and $m > 0$, the Strip-Tiling Algorithm is a stable policy for the DTRP and the resulting system time T_{STA} satisfies:

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$$\lim_{\lambda\to+\infty}\frac{T_{\mathrm{DTRP,m}}}{\lambda^2}\leq\lim_{\lambda\to+\infty}\frac{T_{\mathrm{BTA}}}{\lambda^2}\leq 70.5464\,\frac{\rho W H}{m^3}\left(1+\frac{7}{3}\pi\frac{\rho}{W}\right)^3$$

The achievable performance of the Strip-Tiling Algorithm provides a constant-factor approximation to the lower bound established in Theorem 1.

VI. CONCLUSIONS

In this paper, the TSP for vehicles that follow paths of bounded curvature in the plane is simulated on developed algorithm. For the worstcase and the stochastic settings, obtained upper bounds that are within a constant factor of the lower bound; the upper bounds are constructive in the sense that they are achieved by novel algorithms. It is interesting to compare our results with the Euclidean setting (i.e., the setting in which vehicle paths do not have curvature constraints). Remarkably, the differences between these various bounds play a crucial role when studying the DTRP; e.g., stable policies exist only when the TSP cost grows strictly sublinearly with n. For the DTRP Future directions of research include finding a single algorithm, which would provide constant factor approximation to the DTSP for the worst case as well as the stochastic setting. It is also interesting to consider the non-uniform stochastic DTSP when the points to be visited are sampled according to a non-uniform probability distribution. Other avenues of future research are to use the tools developed in this paper to study Traveling Salesperson Problems for other dynamical vehicles, study centralized and decentralized versions of the DTRP and general task assignment and surveillance problems for multi-Dubins (and other dynamical) vehicles.

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